EE 362: Signals and Systems Name:----------

Spring 2019

Homework #14 (Will not be collected)

- Circle or box your answer (don't forget units).
- **Show** your steps.
- Use 8.5" x 11" paper, and trim off any ragged edges.
- **Staple** your work together and write your name neatly.

1- Work out problem 7.8 from your textbook (Edward W. Kamen and Bonnie S. Heck, Fundamentals of Signals and Systems Using the Web and MATLAB, 3rd Ed, Prentice Hall, 2006).
2- Work out problem 7.9 from your textbook (Edward W. Kamen and Bonnie S. Heck, Fundamentals of Signals and Systems Using the Web and MATLAB, 3rd Ed, Prentice Hall, 2006).
3- Work out problem 7.13, parts (a) and (b) from your textbook (Edward W. Kamen and Bonnie S. Heck, Fundamentals of Signals and Systems Using the Web and MATLAB, 3rd Ed, Prentice Hall, 2006).
4- Work out problem 7.17 part (a) from your textbook (Edward W. Kamen and Bonnie S. Heck, Fundamentals of Signals and Systems Using the Web and MATLAB, 3rd Ed, Prentice Hall, 2006).
5- Work out problem 7.18 part (a) from your textbook (Edward W. Kamen and Bonnie S. Heck, Fundamentals of Signals and Systems Using the Web and MATLAB, 3rd Ed, Prentice Hall, 2006).
6- Work out problem 7.32 from your textbook (Edward W. Kamen and Bonnie S. Heck, Fundamentals of Signals and Systems Using the Web and MATLAB, 3rd Ed, Prentice Hall, 2006).
7- Work out problem 7.35 from your textbook (Edward W. Kamen and Bonnie S. Heck, Fundamentals of Signals and Systems Using the Web and MATLAB, 3rd Ed, Prentice Hall, 2006).
8- Work out problem 7.39 from your textbook (Edward W. Kamen and Bonnie S. Heck, Fundamentals of Signals and Systems Using the Web and MATLAB, 3rd Ed, Prentice Hall, 2006).
9- Work out problem 7.41 from your textbook (Edward W. Kamen and Bonnie S. Heck, Fundamentals of Signals and Systems Using the Web and MATLAB, 3rd Ed, Prentice Hall, 2006).

7.6 (d) $X(z) = \frac{az}{z^2 + a^2}$, $x_{sum} = \frac{a}{1+a^2}$

7.7 $cp^n + \bar{c}\bar{p}^n = |c|\sigma^n e^{j(\Omega n + \angle c)} + |c|\sigma^n e^{-j(\Omega n + \angle c)}$

$= 2|c|\sigma^n \left(\frac{e^{j(\Omega n + \angle c)} + e^{-j(\Omega n + \angle c)}}{2} \right)$

$= 2|c|\sigma^n \cos(\Omega n + \angle c)$

7.8 $X(z) = -\frac{1}{2} + \frac{2}{z-1} = -\frac{1}{2} + z^{-1}\frac{2z}{z-1}$

So, $x[n] = -\delta[n-1] + 2u[n-1]$ and

$x[0] = 0$, $x[1] = 1$, $x[10{,}000] = 2$

7.9 (a) Using Table 7.3, $x[n] = \sin\left(\frac{\pi n}{2}\right)$, $n \geq 0$

(b) $X(z) = z\left[\frac{z}{z^2+1}\right]$, where $\sin\frac{\pi}{2}n \longleftrightarrow \frac{z}{z^2+1}$

By left shift property, $x[n] = \sin\left[\frac{\pi}{2}(n+1)\right]$, $n \geq 0$

which reduces to $x[n] = \cos\frac{\pi}{2}n$, $n \geq 0$

(c) $X(z) = z^{-1}\frac{z}{z^2+1} + \frac{\frac{1}{2}}{z-1} - \frac{\frac{1}{2}}{z+1}$

$x[n] = \sin\left(\frac{\pi}{2}(n-1)\right)u[n-1] + \frac{1}{2}\left(1 + (-1)^n\right)u[n-1]$

$= \left[-\cos\left(\frac{\pi}{2}n\right) + \frac{1}{2}\left(1 + (-1)^n\right)\right]u[n-1]$

(d) $z\frac{1}{z^2-1} \longleftrightarrow \frac{1}{2}\left(1 + (-1)^{n+1}\right)u[n]$, so

$x[n] = \cos\frac{\pi n}{2} + \frac{1}{2}\left(1 - (-1)^n\right)$, $n \geq 0$

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7.9 (e) $X(z) = \frac{z^2}{z^2+1} - \frac{1}{z^2+1}$

$x[n] = \left(\cos\left(\frac{\pi}{2}n\right)\right)u[n] + \left(\cos\left(\frac{\pi n}{2}\right)\right)u[n-1]$

$= \delta[n] + \left(2\cos\left(\frac{\pi n}{2}\right)\right)u[n-1]$

(f) $\frac{X(z)}{z} = -\frac{2}{z} + \frac{3/2}{z-1} + \frac{1/4(1+j3)}{z-j} + \frac{1/4(1-j3)}{z+j}$

$x[n] = -2\delta[n] + \frac{3}{2} + \frac{\sqrt{10}}{2}\cos\left(\frac{\pi n}{2} + 1.249\right)$ for $n \geq 0$

(g) $\frac{X(z)}{z} = \frac{-2}{z} + \frac{3/2}{z-1} + \frac{1/4(1+j)}{z-j} + \frac{1/4(1-j)}{z+j}$

$x[n] = -2\delta[n] + \frac{3}{2} + \frac{\sqrt{2}}{2}\cos\left(\frac{\pi n}{2} + \frac{\pi}{4}\right)$ for $n \geq 0$

Also, note that $X(z) = \underbrace{\frac{z+2}{(z-1)(z^2+1)}}_{X(z) \text{ in part (f)}} + \underbrace{\frac{z}{z^2+1}}_{\sin\frac{\pi n}{2}}$

(h) $nx[n] \longleftrightarrow -z\frac{d}{dz}\left[\ln\left(\frac{2z-1}{2z}\right)\right] = -z\left(\frac{2z}{2z-1}\right)\frac{1}{2z^2}$

$= \frac{-0.5}{z-0.5}$

Hence, $nx[n] = -0.5(0.5)^{n-1}u[n-1] = -(0.5)^n u[n-1]$

and therefore $x[n] = -\frac{(0.5)^n}{n}u[n-1]$

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7.9 continued

The computation of the inverse z-transform using the MATLAB command `iztrans` is as follows:

(a)

```
syms X x z
X=z/(z^2+1);
x=iztrans(X)
```

yields `x =`

```
sin(1/2*pi*n)
```

(b)

```
syms X x z
X=z^2/(z^2+1);
x=iztrans(X)
```

yields `x =`

```
cos(1/2*pi*n)
```

(c) The Symbolic Math Toolbox gives

```
iztrans(1/(z^2+1)) = 0
```

which is incorrect. Hence, applying the command `iztrans` to the $X(z)$ given in Part c results in an incorrect answer.

(d)

```
syms X x z
X=z^2/(z^2+1)+z/(z^2-1);
x=iztrans(X)
```

yields `x =`

```
cos(1/2*pi*n)-1/2*(-1)^n+1/2
```

which is the correct solution.

(e) The Symbolic Math Toolbox gives `iztrans((z^2-1)/(z^2+1))= cos(1/2*pi*n)`, which is incorrect.

(f) The Symbolic Math Toolbox gives a strange solution for this problem.

(g) The Symbolic Math Toolbox gives a strange solution for this problem.

(h) The Symbolic Math Toolbox cannot be applied to $X(z)$ due to the presence of the ln function.

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7.13 (a) $X(z) = \frac{z}{z-1} + 3z^{-1}$, $V(z) = \frac{z^{-1}}{z-1}$

$$X(z)V(z) = \frac{1}{(z-1)^2} + \frac{3z^{-2}}{z-1} \longleftrightarrow (n-1)\,u[n-1] + 3u[n-3]$$

(b) $X(z) = \frac{z}{z-1}$, $V(z) = \frac{z}{(z-1)^2}$, From Table 7.3,

$$X(z)V(z) = \frac{z^2}{(z-1)^3} \longleftrightarrow \frac{1}{2}n(n+1)u[n]$$

(c) $X(z) = \frac{z}{z^2+1}$, $v[n] = e^{-(n-2)}e^{-2}u[n-2] \longleftrightarrow \frac{e^{-2}}{z(z-e^{-1})}$

$$X(z)V(z) = \frac{e^{-2}}{(z^2+1)(z-e^{-1})} = \frac{\frac{e^{-2}}{e^{-2}+1}}{z-e^{-1}} + \frac{-\frac{e^{-2}}{e^{-2}+1}z}{z^2+1} + \frac{\frac{-e^{-3}}{e^{-2}+1}}{z^2+1}$$

$$z^{-1}\,\frac{\frac{e^{-2}}{e^{-2}+1}z^2}{z^2+1} \longleftrightarrow \frac{-e^{-2}}{e^{-2}+1}\cos\left(\frac{(n-1)\pi}{2}\right)u[n-1]$$

$$= -\frac{e^{-2}}{e^{-2}+1}\sin\left(\frac{n\pi}{2}\right)u[n-1]$$

$$z^{-1}\,\frac{\frac{-e^{-3}}{e^{-2}+1}z}{z^2+1} \longleftrightarrow \frac{-e^{-3}}{e^{-2}+1}\sin\frac{(n-1)\pi}{2}\,u[n-1] = \frac{e^{-3}}{e^{-2}+1}\cos\frac{n\pi}{2}\,u[n-1]$$

$$x[n]*v[n] = \frac{e^{-2}}{e^{-2}+1}\left[e^{-(n-1)} - \sin\frac{n\pi}{2} + e^{-1}\cos\frac{n\pi}{2}\right]u[n-1]$$

(d) $x[n]*v[n] = u[n-1]*v[n]+v[n]$ so we only need to compute $u[n-1]*v[n] \longleftrightarrow$

$$\frac{1}{z-1}\left(\frac{z}{z-e^{-1}} - \frac{2e^{-4}z^{-1}}{z-e^{-2}}\right) = \frac{z}{(z-1)(z-e^{-1})} - \frac{2e^{-4}z^{-1}}{(z-1)(z-e^{-2})}$$

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7.16 $X(z) = z^{-1} + \frac{1}{2}z^{-2}$

$$Y(z) = H(z)X(z) = z^{-2} + z^{-3} + \frac{7}{12}z^{-4} + \frac{13}{6}z^{-5} + 4z^{-6} + \frac{3}{2}z^{-7}$$

Thus, $y[2] = 1$, $y[3] = 1$, $y[4] = \frac{7}{12}$, $y[5] = \frac{13}{6}$, $y[6] = 4$,

$y[7] = \frac{3}{5}$, $y[n] = 0$ for all other n

7.17 $Y(z) + 0.7\left(z^{-1}Y(z) + y[-1]\right) = \frac{z}{z-1}$

$$Y(z) = \frac{z^2}{(z-1)(z+0.7)} - \frac{0.7z}{z+0.7}$$

$$= z\left[\frac{10}{17}\frac{1}{z-1} + \frac{7}{17}\frac{1}{z+.7}\right] - \frac{0.7z}{z+0.7}$$

$$= \frac{10}{17} - \frac{49}{170}(0.7)^n \quad \text{for } n \geq 0$$

```
% program to compare results for n = 0 to 10
num = [1 0];
den = [1 .7];
zi = -.7;
y = filter(num,den,ones(1,11),zi)  % MATLAB computation
n = 0:10;
y = 10/17-49/170*(-.7).^n   % analytical expression
```

The values for y from the MATLAB simulation and the values for y from the analytical expression are both given by

```
y =

  Columns 1 through 7

    0.3000    0.7900    0.4470    0.6871    0.5190    0.6367    0.5543

  Columns 8 through 11

    0.6120    0.5716    0.5999    0.5801
```

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7.18 $Y(z) - 0.2(z^{-1}Y(z) + y[-1]) - 0.8(z^{-2}Y(z) + y[-2] + z^{-1}y[-1]) = 0$

$$Y(z) = \frac{1 + 0.8z^{-1}}{1 - 0.2z^{-1} - 0.8z^{-2}} = \frac{z^2 + 0.8z}{z^2 - 0.2z - 0.8} = \frac{z}{z-1}$$

```
num = [0 0 0];
den = [1 -.2 -.8];
zi = [1 .8];
y = filter(num,den,zeros(1,11),zi)  % MATLAB simulation
```

The result below gives y[n] for n = 0 to 10 which matches the analytical result.

```
y =

     1     1     1     1     1     1     1     1     1     1     1
```

7.19 (a) $H(z) = \frac{2z-1}{z^2+1} = 2\frac{z}{z^2+1} - z^{-1}\frac{z}{z^2+1}$

$h[n] = 2\sin\frac{\pi}{2}n - \left(\sin\frac{\pi}{2}(n-1)\right)u[n-1]$

$= 2\sin\frac{\pi n}{2} + \left(\cos\frac{\pi n}{2}\right)u[n-1]$, $n \geq 0$

(b) $Y(z) = \frac{(2z-1)z}{(z^2+1)(z-1)} = z\left[\frac{3/2 - 1/2\,z}{z^2+1} + \frac{1/2}{z-1}\right]$

$y[n] = \frac{1}{2} + \frac{3}{2}\sin\frac{\pi n}{2} - \frac{1}{2}\cos\frac{\pi n}{2}$, $n \geq 0$

(c) $Y(z) = z\,\frac{-2z^2+3z+5}{(z-2)(z^2+1)} = \left[\frac{3/5}{z-2} + \frac{-13/5\,z - 11/5}{z^2+1}\right]z$

$y[n] = \frac{3}{5}(2^n) - \frac{13}{5}\cos\frac{\pi n}{2} - \frac{11}{5}\sin\frac{\pi n}{2}$, $n \geq 0$

(d) $y[n] = 0$ for all n and thus $x[n] = 0$, $n \geq 0$

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7.31 (b) continued

Finally, $Y(z) = Q_1(z) + 5z\,Q_1(z) + Q_2(z) + X(z)$

$$= \frac{5z+1}{z+2}X(z) + \left[\frac{2}{z(z+2)} + 1\right]X(z)$$

$$= \underbrace{\frac{6z^2+3z+2}{z(z+2)}}_{H(z)}X(z)$$



7.32 (a) Let $q_1[n]$ = output of delay on left, $q_2[n]$ = output of delay on right. Then

$$z\,Q_1(z) = -0.02\,Y(z) + X(z)$$
$$z\,Q_2(z) = Q_1(z) + 0.3Y(z) + 2X(z)$$
$$Y(z) = Q_2(z) + 3X(z)$$

Inserting the first equation into the second, we have

$$z^2Q_2(z) = -0.02\,Y(z) + X(z) + 0.3z\,Y(z) + 2z\,X(z) + 3z^2X(z)$$

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7.32 continued

Solving for $Y(z)$ we have

$$Y(z) = \underbrace{\frac{3z^2+2z+1}{z^2-.3z+.02}}_{=H(z)} X(z)$$

(b) $y[n+2] - 0.3y[n+1] + 0.02y[n] = 3x[n+2] + 2x[n+1] + x[n]$

(c) $Y(z) = H(z)X(z) = \left(\frac{3z^2+2z+1}{z^2-0.3z+0.02}\right)\frac{4z}{z-1}$

$$Y(z) = \frac{\frac{100}{3}z}{z-1} + \frac{\frac{164}{3}z}{z-0.1} - \frac{76z}{z-0.2}$$

So $y[n] = \frac{100}{3} + \frac{164}{3}(0.1)^n - 76(0.2)^n, \quad n \geq 0$

7.33 $H_2(z) = \frac{z^2+7z+12}{z^2-3z+2} = 6 - \frac{20z}{z-1} + \frac{15z}{z-2}$

Thus, $h_2[n] = 6\delta[n] - 20 + 15(2)^n, \quad n \geq 0$

7.34 (a) $H(z) = \frac{H_1(z)}{1 + H_1(z)H_2(z)} = \frac{z^2-8z}{z^2+2z-8}$

$$\frac{H(z)}{z} = \frac{2}{z+4} - \frac{1}{z-2}$$

So, $h[n] = 2(-4)^n - 2^n, \quad n \geq 0$

(b) $\frac{Y(z)}{z} = \frac{1.6}{z+4} - \frac{2}{z-2} + \frac{1.4}{z-1}$

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7.35 (a) $Y(z) = X(z) + 0.5z^{-1}Y(z) + 0.25z^{-2}Y(z)$

$$Y(z) = \frac{1}{1-.5z^{-1}+.25z^{-2}}X(z) = \underbrace{\frac{z^2}{z^2-.5z+.25}}_{H(z)}X(z)$$

(b) Want $W(z)H(z)=1$, so

$$W(z) = \frac{1}{H(z)} = \frac{z^2-.5z+.25}{z^2} = 1-0.5z^{-1}+0.25z^{-2}$$

7.36 (a) $$H(z) = \frac{2z+1}{(z+1)(z+2)} = \frac{-1}{z+1} + \frac{3}{z+2}$$

$$= z^{-1}\left[\frac{-z}{z+1} + \frac{3z}{z+2}\right]$$

$$h[n] = -(-1)^{n-1}u[n-1] + 3(-2)^{n-1}u[n-1]$$

(b) $y[n+2] + 3y[n+1] + 2y[n] = 2x[n+1] + x[n]$

(c) $$Y(z) = \frac{2z+1}{(z+1)(z+2)}\frac{z}{(z-1)} = \frac{-.5}{z+1} + \frac{2}{z+2} + \frac{.5}{z-1}$$

$$y[n] = \left[-.5(-1)^{n-1} + 2(-2)^{n-1} + .5\right]u[n-1]$$

(d) $$Q(z) = \frac{-2z}{z+1} + \frac{2z+1}{z+1}\frac{z}{z+1} = \frac{3z}{(z+1)(z-1)}$$

$$y[n+1] + 2y[n] = q[n]$$

$$Y(z) = \frac{1}{z+2}Q(z) + \frac{y[0]z}{z+2} = \frac{1}{z+2}\frac{3z}{(z+1)(z-1)} + \frac{z(-4)}{z+2}$$

$$\frac{Y(z)}{z} = \frac{-3}{z+2} - \frac{1.5}{z+1} + \frac{0.5}{z-1}$$

$$y[n] = -3(-2)^n - 1.5(-1)^n + 0.5, \quad n \geq 0$$

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7.37 $H(z) = \frac{Y(z)}{X(z)} = \frac{\frac{z}{z-.5}}{1+\frac{2}{z-1}} = \frac{z(z-1)}{(z-.5)(z+1)}$

poles are $z = .5$ and -1, so the system is marginally stable

7.38 Poles are $z = 0.1, 0.2$ and thus the system is stable.

7.39 $(z^2-z+1)\,Y(z) = (z^2-z)\,X(z)$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^2-z}{z^2-z+1}$$

poles are $0.5 \pm j\,0.866$ which have magnitude of 1 so it is marginally stable

7.40 (a) $\sum_{n=0}^{\infty} |h[n]| = \sum_{n=0}^{9} \left|\sin\frac{n\pi}{6}\right| < \infty$

so the system is BIBO stable

(b) $\sum_{n=0}^{\infty} |h[n]| = \sum_{n=1}^{\infty} \frac{1}{n} = \infty$

So the system is not BIBO stable

(c) $\sum_{n=0}^{\infty} |h[n]| = \sum_{n=1}^{\infty} \frac{1}{n^2} < \infty$ so the system is BIBO stable

(d) $\sum_{n=0}^{\infty} |h[n]| = \sum_{n=0}^{\infty} \left|e^{-n}\sin\left(\frac{n\pi}{6}\right)\right| \le \sum_{n=0}^{\infty} e^{-n} = \frac{1}{1-e^{-1}} < \infty$

thus the system is BIBO stable

7.41

(a) Using the MATLAB command r=roots([1 1.5 .5]) gives r = -1.0000 and -0.5000, and thus the system is not stable since there is a pole on the unit circle.

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7.41 continued

(b) The MATLAB command `r=roots([1 1 -.5 .5])` gives

```
r =   -1.5370
       0.2685 + 0.5032i
       0.2685 - 0.5032i
```

Hence there is a pole outside of the closed unit disc and the system is unstable.

(c) The MATLAB command `r=roots([1 1 -.5 .5])` gives

```
r =    0.0935 + 0.7254i
       0.0935 - 0.7254i
      -0.1869
```

The magnitude of the poles are `abs(r)` =0.7314, 0.7314, and 0.1869. All three poles are less than 1 in magnitude, and therefore, the system is stable.

7.42 (a) $H(\Omega) = \dfrac{e^{j\Omega}}{e^{j\Omega}+0.5}$

$H(3) = 1.96 e^{j8^\circ}$ so

$y_{ss}[n] = 9.8\cos(3n+8^\circ)$, $n \geq 0$

Now $\dfrac{Y(z)}{z} = \dfrac{5(z^2-\cos 3z)}{(z+.5)(z^2-2(\cos 3)z+1)}$

$$\frac{Y(z)}{z} = \underbrace{-\frac{4.71}{z+.5}}_{\frac{Y_{tr}(z)}{z}} + \frac{Y_{ss}(z)}{z}$$

Hence, $y_{tr}[n] = -4.71(-.5)^n u[n]$

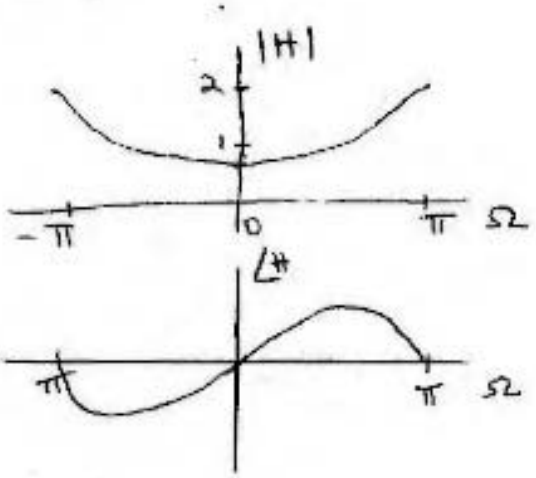
(b) $|H(\Omega)| = \dfrac{1}{\sqrt{\cos\Omega + 1.25}}$

min for $\Omega = 0$, $|H(0)| = 0.67$

max for $\Omega = \pi$, $|H(\pi)| = 2$

$\angle H(\Omega) = \Omega - \tan^{-1}\dfrac{\sin(\Omega)}{.5+\cos\Omega}$

$\angle H(0) = 0$, $\angle H(\pi) = 0$, $\angle H(\tfrac{\pi}{2}) = 27^\circ$



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